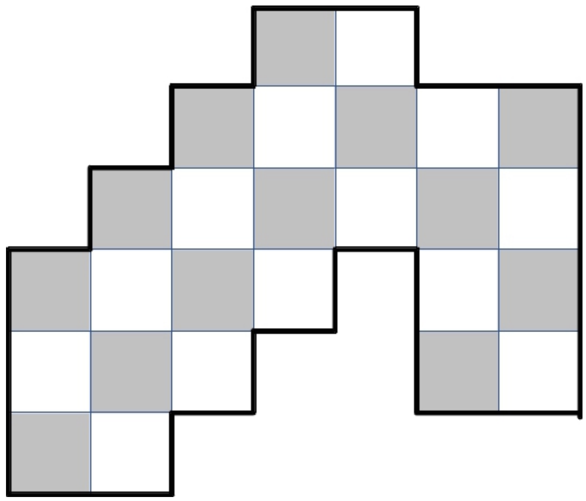
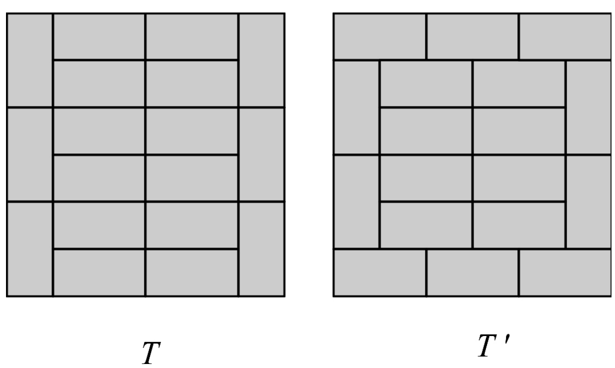
Flip Dynamics (Exercises)

Exercise 1. Compute the minimal tiling of the domain below, using the algorithm given in the lecture.



Exercise 2. What is the minimal number of necessary local flips to pass from a tiling below to an another one ? How many flips will be done in the vertex in the center ?

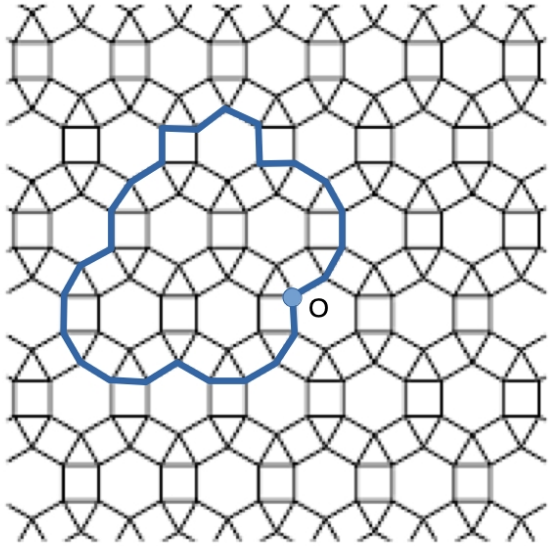


Exercice 3. a) In the given tessellation, direct edges in such a way that the contour of each square is done clockwise. What about the contour of hexagons and triangles ?

b) Attribute a positive height w(e) on each undirected edge of D, in such a way that the sum of weights of edges of any cell is equal to 6.

We define the signed weight $w(v, v')$ of the directed arc (v, v') as

- $w(v, v') = w(\{, v, v'\})$ if (v, v') belongs to the direction given is (a),
- $w(v, v') = -w(\{, v, v'\})$ otherwise.



This definition is canonically extended to paths, by addition

c) Prove that if a domain contains 50% of square cells, then the signed weight of its boundary path is null.

d) We want to study tilings whose tiles are formed from two cells sharing an edge (i.e. either a hexagon and a square, or a triangle and a square). Draw such a tiling T of the domain D given.

d) Using weights on edges, define a height function which allows to encode such tilings. Give it for the tiling T.

e) Is there an interior local maximum ? if yes, change the tiling around it by a local flip. Repeat it until there is no more interior local maximum.

f) What is the tiling obtained ? How can it be constructed directly ?

Exercice 4 : Describe the shape of the Cayley graph corresponding to the presentation:

$$< a, b \,|\, a^5, b^2, (ab)^3 >$$

Exercice 5 : The goal of this exercice is to study the tiling space of a domain D, where tiles are 3×1 and 1×3 rectangles, (covering three cells of the square grid).

a) Check that the group

$$A =< a, b \,|\, a^3, b^3 >$$

can be used as an auxiliary group in order to study these tilings,

b) Describe the shape of the Cayley graph linked to the presentation above.

c) Introduce a height function on this Cayley graph

d) Let T be a minimal tiling (i.e. there is no tiling $T' \neq T$ such that $h_{T'} \leq h_T$), and let $M = \max\{h_T(v), v \in D\}$. What can we say about the position of a vertex v such that $h_T(v) = M$?

e) Deduce the uniqueness of the minimal tiling T, and therefore, the connectivity of the space of tilings, using flips which have been introduced in d)