Exercice 1 [Non-existence for the eigenvalue problem] Consider the growth fragmentation equation with the growth rate \( \tau(x) = 1 \), the division rate
\[
B(x) = \begin{cases} 
0, & 0 \leq x < 1, \\
b, & 1 \leq x < 2, \\
0, & 2 \leq x,
\end{cases}
\]
for some \( b > 0 \), and the mitosis kernel \( \kappa(z) = \delta(z - 1/2) \). Prove that for \( b < \ln(2) \) there is no solution to the eigenproblem.

Exercice 2 [Monotony of the Malthus parameter] Consider the growth fragmentation equation with the two sets of parameters \((\tau = \tau_1, B = \tau_1 \beta, k)\) and \((\tau = \tau_2, B = \tau_2 \beta, k)\). We assume that
\[
\tau_1(x) \leq \tau_2(x), \quad x \in \mathbb{R}^+
\]
Prove that the associated Malthus parameters \( \lambda_1 \) and \( \lambda_2 \) verify
\[
\lambda_1 \leq \lambda_2.
\]

Exercice 3 [Asymptotic behaviour for the pure fragmentation equation] Consider the pure fragmentation equation
\[
\begin{cases} 
\frac{\partial}{\partial t} u(t, x) = -B(x)u(t, x) + 2 \int_{x}^{\infty} B(y) \kappa \left( \frac{x}{y} \right) u(t, y) \frac{dy}{y}, \\
u(0, x) = u_0(x), \quad u_0 \in L^1(\mathbb{R}^+),
\end{cases}
\]
where \( B(x) = \alpha x^\gamma \) for \( \alpha > 0, \gamma > 0, \kappa \in \mathcal{M}^+(\mathbb{R}^+) \) such as
\[
\int_0^1 \kappa(z)dz = 1, \quad \int_0^1 z\kappa(z)dz = \frac{1}{2}.
\]

a) Show that fragmentation equation can be turned into a growth-fragmentation equation with the change of variables \((t = t, y = t^{1/\gamma} x)\).

b) Deduce the asymptotic behaviour of the solution \( u \) to \((1)\).