Bratteli-Vershik Models of Cantor Minimal Systems

Exercises

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(1) For the Bratteli diagram (B, V) prove that X_B (the set of all infinite paths) with the topology generated by cylinders (finite paths) is a second countable, totally disconnected, compact and Hausdorff space.

(2) Prove that the Vershik system on a simple ordered Bratteli diagram is homeomorphism and (X_B, T_B) is minimal.

(3) A dynamical system is called *essentially minimal* if it has a *unique minimal subsystem*. Try to construct a Bratteli diagram (B, V) with an ordering on it such that (X_B, T_B) is essentially minimal.

(4) Show that every Bratteli diagram has an equivalent diagram in which between v_0 and any vertex at level V_1 there is a single edge.

(5) Consider the minimal system (X, T) on the Cantor set and a clopen subset $U \subset X$. For every $x \in X$ let

$$n(x) = \inf\{n \in \mathbb{N} : \; T^n x \in U\} > 0.$$

Prove that for U, $n(x)$ has a uniform bound on X.

(6) Suppose that (X_B, T_B) is a Vershik system on the simple ordered Bratteli diagram $(B, V, \leq)$. If U is a clopen subset of X_B, how one can show the induced system $(U, (T_B)_U)$ on $(B, V, \leq)$.

(7) Prove that different levels of one or two towers of a Kakutani-Rokhlin partition are disjoint.

(8) If (X, T) is expansive then

$$\exists\ k_0 \text{ such that } \forall k > k_0 : \quad (X, T) \text{ is conjugate to } (Y_k, S)$$

where (Y_k, S), $k \in \mathbb{N}$ are the factors of (X, T) coming up from the truncation maps on the Bratteli-Vershik model of (X, T).

(9) Suppose that the minimal Cantor system (X, T) has the Bratteli-Vershik model $(B, V, \leq)$ and let $(B', V', \leq')$ be another simple ordered Bratteli diagram that $V = V'$, $E = E'$ and the orderings on $r^{-1}(v)$ for all $v \in V_n = V'_n$, $n \in \mathbb{N}$ are the same except at one level k_0. We have seen that the Vershik system on $(B', V', \leq')$ is orbit equivalent to the one on $(B, V, \leq)$. show that the two cocycle maps are bounded. (hint. one may use K-R towers.)